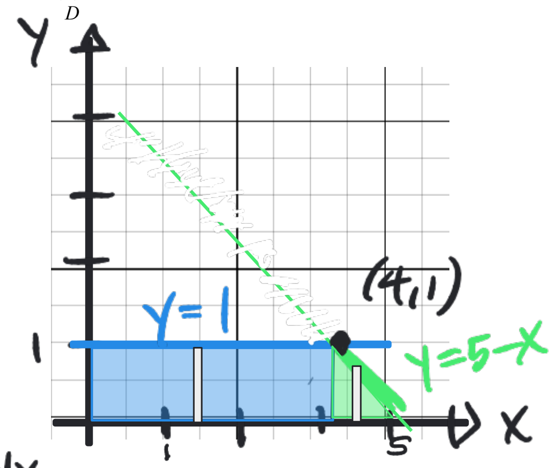
I prefer dxdy for two reasons. It is only one integral, but also, if y is first, we will get a y^4 which then is evaluated at $5-x \Rightarrow (5-x)^4$

Classwork 10/15

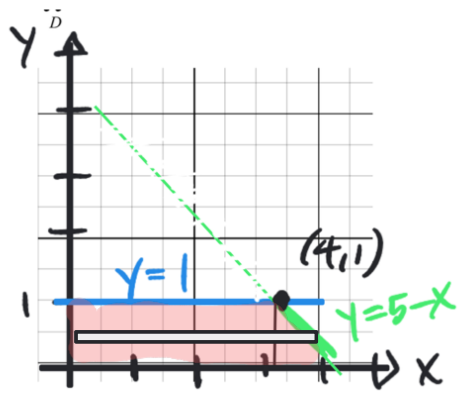
(1). Set up the integral for both orders of integration. Then evaluate the double integral using the easier order and explain $\boxed{\text{why it is easier.}}$

$\iint_D y^3\, dA$, where D is the region bound by $y=1$, $x+y=5$, and the coordinate axes.



dydx need to split

$$\boxed{\iint_D y^3\, dA = \int_0^4\int_0^1 y^2\,dy\,dx + \int_4^5\int_0^{5-x} y^3\,dy\,dx}$$



dxdy no need to split

$$\boxed{\int_0^1\int_0^{5-y} y^3\,dx\,dy} = \int_0^1 xy^3\Big]_0^{5-y} dy = \int_0^1 (5-y)y^3\,dy$$

$$= \int_0^1 (5y^3 - y^4)\,dy = \frac{5}{4}y^4 - \frac{1}{5}y^5\Big]_0^1$$

$$= \frac{5}{4} - \frac{1}{5} = \frac{24}{20} = \boxed{\frac{6}{5}}$$

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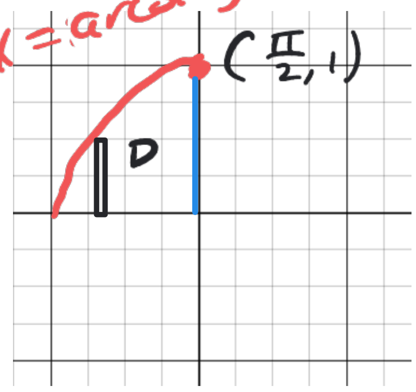
(2) Compute $\displaystyle\int_0^1 \int_{\arcsin(y)}^{\pi/2} \cos x\sqrt{1+\cos^2 x}\; dx\, dy$

You may want to reverse the order of integration.

y = sin x
x = arcsin y



x = arcsin y

$\Rightarrow$ y = sin x for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\boxed{\int_0^{\pi/2}\int_0^{\sin x} \cos x\sqrt{1+\cos^2 x}\; dy\, dx}$$

$$= \int_0^{\pi/2} y \cos x\sqrt{1+\cos^2 x}\Big]_0^{\sin x} dx = \int_0^{\pi/2} \sin x\cos x\sqrt{1+\cos^2 x}\, dx$$

$u = 1+\cos^2 x$

$du = -2\cos x \sin x\, dx$

$$-\frac{1}{2}\int_2^1 \sqrt{u}\, du = -\frac{1}{3}u^{3/2}\Big]_2^1$$

$$= -\frac{1}{3} + \frac{1}{3}\cdot 2^{3/2}$$

$$= \boxed{\frac{1}{3}(2^{3/2}-1)}$$

note: there are other ways to do this integral